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Optical Solitons with Schrödinger-Hirota Equation for Kerr Law Nonlinearity

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Abstract:
In this paper, The Schrödinger–Hirota equation will be solved by the Sech, Tanh, and Csch methods for bright, dark, singular 1-soliton solution. Kerr law nonlinearity media is studied. Soliton solutions obtained will be important for the conservation laws for dispersive optical solitons.

Keywords: Nonlinear PDEs, The Schrödinger–Hirota equation, Exact Solutions, Sech –Tanh Csch function methods,

1. INTRODUCTION

All optical communications are being used for trans-continental and trans-oceanic data transfer, through long-haul optical fibers, at the present time. There are various aspects of soliton communication that still needs to be addressed. One of the features is the dispersive optical solitons. In presence of higher order dispersion terms, soliton communications are sometimes a hindrance as these dispersion terms produce soliton radiation. Nonlinear evolution equations have a major role in various scientific and engineering fields, such as optical fibers. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. In recent years, quite a few methods for obtaining explicit traveling and solitary wave solutions of nonlinear evolution equations have been proposed. A variety of powerful methods has been used to solve different types of nonlinear systems of PDEs.[1-14]

The nonlinear Schrödinger–Hirota equation governs the propagation of optical solitons in a dispersive optical fiber is a very important equation in the area of theoretical and mathematical physics. This paper is going to take a look at the bright, dark, singular for Kerr law nonlinearity media.

2. THE TRAVELING WAVE SOLUTION

Consider the nonlinear partial differential equation in the form

$$F(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{xy}, u_{yy}, \dots \dots \dots \dots) = 0 \tag{1}$$

where u(x, y, t) is a traveling wave solution of nonlinear partial differential equation Eq. (1). We use the transformations,

$$u(x, y, t) = f(\xi) \tag{2}$$

where $\xi = x + y - \lambda t$ This enables us to use the following changes:

$$\frac{\partial}{\partial t}(.) = -\lambda \frac{d}{d\xi}(.)\ ,\ \frac{\partial}{\partial x}(.) = \frac{d}{d\xi}(.)\ ,\ \frac{\partial}{\partial y}(.) = \frac{d}{d\xi}(.) \tag{3}$$

Using Eq. (3) to transfer the nonlinear partial differential equation Eq. (1) to nonlinear ordinary differential equation

$$Q(f, f', f'', f''', \dots \dots \dots \dots \dots .) = 0 \tag{4}$$

The ordinary differential equation (4) is then integrated as long as all terms contain deriva0tives, where we neglect the integration constants.

3. GOVERNING EQUATIONS

Nonlinear Schrödinger's equation (NLSE) is governed the dynamics of soliton propagation through optical fibers. Occasionally, when the group velocity dispersion (GVD) is small, one needs to supplement with third order dispersion (3OD) to maintain the balance between dispersion and nonlinearity so that soliton propagation stays stable. Thus, the dimensionless form of NLSE with 3OD is given by [3, 4, 5] ;

$$i\, u_t + \frac{1}{2} u_{xx} + |u|^2\ u = -i\, \lambda\, u_{xxx} \tag{5}$$

Here λ is the coefficient of 3OD. The first term on the left hand side is the linear temporal evolution, while the second term represents GVD. Also, on left side, the third term accounts for Kerr law nonlinearity and the first term accounts for temporal evolution. For studying this model from a mathematical perspective the following Lie transform is considered.[15, 16]

$$q = u - 3i\, \lambda \left[u_x + 2\, u \int_{-\infty}^{x} |u|^2\ \ d\xi\right] \tag{6}$$

Then Eq.(8) is transformed to:

$$i\, q_t + \frac{1}{2} q_{xx} + |q|^2\ q + i\, \lambda\, (\, q_{xxx} + 6\, |q|^2\ q_x\,) = 0 \tag{7}$$

without considering higher order terms. This is Schrödinger-Hirota equation (SHE) with Kerr law nonlinearity that models propagation of dispersive solitons through optical fibers. Next, rewriting this equation with arbitrary coefficients, one obtains [15, 16]

$$i\, q_t + \ a\, q_{xx} + c\, |q|^2\ q + i\ \ (\gamma\, q_{xxx} + \sigma\, |q|^2\ q_x\,) = 0 \tag{8}$$

Where, σ is the coefficient of nonlinear dispersion. Now, it was reported earlier during 2012 that GVD alone makes NLSE ill-posed.[17] Therefore an additional dispersion term, known as spatio-temporal dispersion, was suggested to be included[17]. This leads to SHE with spatio-temporal dispersion being given by

$$i\, q_t + a\, q_{xx} + b\, q_{xt} + c\, |q|^2\, q + i\, (\gamma\, q_{xxx} + \sigma\, |q|^2\, q_x) = 0 \tag{9}$$

where the coefficient of b is spatio-temporal dispersion. With perturbation terms Eq.(19) modifies to[2]:

$$i\, q_t + a\, q_{xx} + b\, q_{xt} + c\, |q|^2\, q + i\, (\gamma\, q_{xxx} + \sigma\, |q|^2\, q_x) - i\, \alpha\, q_x + i\, \lambda\, (|q|^2\, q)_x - i\, v\, (|q|^2)_x\, q = 0 \tag{10}$$

This paper will analyze Eq. (10). The study consider two cases where Kerr law and its generalized the power law nonlinearity.

The starting hypothesis for solving (10) by the aid of traveling waves is given by[2]

Introduce the transformations

$$q(x,t) = e^{i\,\theta(x,t)}\, p(\xi), \tag{11}$$

$$\theta = -k\, x + \omega\, t + \epsilon_0\,, \quad \xi = (x - v\, t + \chi) \tag{12}$$

where k, ω, v, and χ are real constants. The parameter v represents the soliton velocity in (12).

Substituting (11-12) into Equation (10) and decomposing into real and imaginary parts leads to:

$$[\omega + k\alpha + k^3\gamma + a\, k^2 - \omega b\, k]p - (k\sigma + c - 3\, k\, \lambda - 2\, kv)p^3 + [v\, b - 3k\gamma - a\,]\, p'' = 0 \tag{13}$$

$$[b\, kv - v - \alpha + b\, \omega - 2a\, k - 3\, k^2\gamma]\, p' + [\, \sigma - 3\lambda - 2v\,]\, p^2 p' + \gamma\, p''' = 0 \tag{14}$$

Integrating Eq.(14) with zero constant,

$$3[2a\, k + 3\, k^2\gamma - b\, kv + v + \alpha - b\, \omega]\, p + [\, \sigma - 3\lambda - 2v\,]\, p^3 - 3\, \gamma\, p'' = 0 \tag{15}$$

Equations (13-15) can be written as:

$$K_1\, p - K_2\, p^3 - K_3\, p'' = 0 \tag{16}$$

$$K_4\, p - K_5\, p^3 - 3\gamma\, p'' = 0 \tag{17}$$

Where:

$$K_1 = [\, \omega + k\alpha + k^3\gamma + a\, k^2 - \omega b\, k] \tag{18}$$

$$K_2 = [\, k\sigma + c - 3\, k\, \lambda - 2\, kv] \tag{19}$$

$$K_3 = [-v\, b + 3\, k\gamma + a\,] \tag{20}$$

$$K_4 = 3[2a\, k + 3\, k^2\gamma - b\, kv + v + \alpha - b\, \omega] \tag{21}$$

$$K_5 = [-\sigma + 3\lambda + 2v\,] \tag{22}$$

4. HYBERBOLIC FUNCTION METHODS

The solutions of many nonlinear equations can be expressed in the form:

4.1 SECH FUNCTION METHOD (Bright Soliton)

$$f(\xi) = A\, sech^{\beta}(\mu\xi)$$

$$f'(\xi) = -A\,\beta\,\mu\, sech^{\beta}(\mu\xi)\,.\,tanh(\mu\xi) \tag{23}$$

$$f''(\xi) = -A\,\beta\,\mu^2\left[(\beta+1)sech^{\beta+2}(\mu\xi) - \beta\, sech^{\beta}(\mu\xi)\right]$$

$$f'''(\xi) = A\,\beta\,\mu^3\left[(\beta+1)(\beta+2)\, sech^{\beta+2}(\mu\xi) - \beta^2\, sech^{\beta}(\mu\xi)\right] tanh(\mu\xi)$$

4.2 TANH FUNCTION METHOD (Dark Soliton)

$$f(\xi) = A\, tanh^{\beta}(\mu\xi)\ \ ,\quad |\xi| \le \frac{\pi}{2\mu}$$

$$f'(\xi) = A\,\beta\,\mu\left[tanh^{\beta-1}(\mu\xi) - tanh^{\beta+1}(\mu\xi))\right] \tag{24}$$

$$f''(\xi) = A\,\beta\,\mu^2\left[(\beta-1)\, tanh^{\beta-2}(\mu\xi) - 2\beta\, tanh^{\beta}(\mu\xi) + (\beta+1)\, tanh^{\beta+2}(\mu\xi)\right]$$

$$f'''(\xi) = A\,\beta\,\mu^3\begin{bmatrix}(\beta-1)(\beta-2)\, tanh^{\beta-3}(\mu\xi) - \{(\beta-1)(\beta-2)+2\beta\} tanh^{\beta-1}(\mu\xi)\\ +\{(\beta+1)(\beta+2)+2\beta\}\, tanh^{\beta+1}(\mu\xi) - (\beta+1)(\beta+2)\, tanh^{\beta+3}(\mu\xi)\end{bmatrix}$$

4.3 CSCH FUNCTION METHOD (Singular Soliton type I)

$$f(\xi) = A\, csch^{\beta}(\mu\xi)$$

$$f'(\xi) = -A\,\beta\,\mu\, csch^{\beta}(\mu\xi)\,.\,coth(\mu\xi) \tag{25}$$

$$f''(\xi) = A\,\beta\,\mu^2\left[(\beta+1)csch^{\beta+2}(\mu\xi) + \beta\, csch^{\beta}(\mu\xi)\right]$$

$$f'''(\xi) = -A\,\beta\,\mu^3\left[(\beta+1)(\beta+2)\, csch^{\beta+2}(\mu\xi) + \beta^2\, csch^{\beta}(\mu\xi)\right] coth(\mu\xi)$$

and so on. We substitute (23) or (24) or (25) into the reduced equation (10), balance the terms of the sech , tanh, and csch functions when (23) or (24) or (25) are used, and solve the resulting system of algebraic equations by using computerized symbolic packages. We next collect all terms with the same power in $sech^{k}(\mu\xi)$ or $tanh^{k}(\mu\xi)$ or $csch^{k}(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknown's A , μ and β, and solve the subsequent system.

5. Application

5.1 Bright Soliton

Seeking the solution by sech function method as in (23)

$$p(\xi) = A\, sech^{\beta}(\mu\, \xi) \tag{27}$$

where A represent the amplitudes of the solitons, μ represents the solitons width.

the system of equations in Eqs. (16) and (17) becomes respectively:

$$K_1 sech^{\beta}(\mu\, \xi) - K_2\, A^2\, sech^{3\beta}(\mu\, \xi) + K_3\, \beta\, \mu^2 \left[(\beta+1) sech^{\beta+2}(\mu\xi) - \beta\, sech^{\beta}(\mu\xi)\right] = 0 \tag{28}$$

$$K_4\, sech^{\beta}(\mu\, \xi) - K_5\, A^2\, sech^{3\beta}(\mu\, \xi) + 3\,\gamma\,\beta\,\mu^2 \left[(\beta+1) sech^{\beta+2}(\mu\xi) - \beta\, sech^{\beta}(\mu\xi)\right] = 0 \tag{29}$$

Equating the exponents and the coefficients of each pair of the sech functions we

find from Eqsn. (28-29)

$$3\beta = \beta + 2 \quad \text{, then } \beta = 1 \tag{30}$$

Thus setting coefficients of Equations (28-29) to zero yields set system of equations:

$$K_1 - K_3\ \mu^2 = 0$$

$$K_4 - 3\,\gamma\ \mu^2 = 0$$

$$2\,K_3\,\mu^2 - K_2\,A^2 = 0$$

$$K_5\,A^2 - 6\,\gamma\ \mu^2 = 0 \tag{31}$$

Solving the system of equations in (33) we get:

<u>Case 1</u> $\mu = \mp\sqrt{\frac{K_1}{K_3}}$ $A = \mp\sqrt{\frac{2\,K_1}{K_2}}$ (32)

<u>Case 2</u> $\mu = \mp\sqrt{\frac{K_1}{K_3}}$ $A = \mp\sqrt{\frac{6\,\gamma}{K_5}\ \frac{K_1}{K_3}}$ (33)

<u>Case 3</u> $\mu = \sqrt{\frac{K_4}{3\,\gamma}}$ $A = \mp\sqrt{2\ \frac{K_3}{K_2}\ \frac{K_4}{3\,\gamma}}$ (34)

<u>Case 4</u> $\mu = \sqrt{\frac{K_4}{3\,\gamma}}$ $A = \mp\sqrt{\frac{2K_4}{K_5}}$ (35)

Where:

$K_i > 0 \;\; i = 1, 2, 3, 4, 5$ (36)

Then :

$p(x,t) = A \;\; sech(\mu \;\; (x - v\,t + \chi))$ (37)

and

$q(x,t) = e^{i\,\theta(x,t)}\, A \;\; sech(\mu \;\; (x - v\,t + \chi))$ (38)

Where the phase component of the soliton is given by

$\theta = -k\,x + \omega\,t + \epsilon_0$ (39)

5.2 Dark Soliton

Seeking the solution by tanh function method

$p(\xi) = \; A\, tanh^{\beta}\,(\mu\,\xi)$ (40)

where A represent the amplitudes of the solitons, μ represents the solitons width.

the system of equations in Eqs. (16) and (17) becomes respectively:

$K_1\,A\,tanh^{\beta}\,(\mu\,\xi) - K_2\,A^3\,tanh^{3\beta}\,(\mu\,\xi) - \; K_3\,A\,\beta\,\mu^2\,\left[(\beta - 1)\,tanh^{\beta\,-\,2}(\mu\xi) - 2\beta\,tanh^{\beta}\,(\mu\xi) + (\beta + 1)\,tanh^{\beta\,+\,2}(\mu\xi)\right] = 0$ (41)

$K_4\,A\,tanh^{\beta}\,(\mu\,\xi) - K_5\,A^3\,tanh^{3\beta}\,(\mu\,\xi) - 3\,\gamma\,A\,\beta\,\mu^2\,\left[(\beta - 1)\,tanh^{\beta\,-\,2}(\mu\xi) - 2\beta\,tanh^{\beta}\,(\mu\xi) + (\beta + 1)\,tanh^{\beta\,+\,2}(\mu\xi)\right] = 0$ (42)

Equating the exponents and the coefficients of each pair of the sech functions we find from (42-43)

$3\beta = \beta + 2$, then $\beta = 1$ (43)

Thus setting coefficients of Equations (43-44) to zero yields set system of equations:

$K_1 + 2\;K_3\;\;\mu^2 \;= 0$

$K_4 + 6\,\gamma\;\;\mu^2 \;\; = 0$

$-K_2\,A^2 - 2\;\;K_3\,\mu^2 = 0$

$-\,K_5\,A^2\; - 6\,\gamma\;\;\mu^2 = 0$ (44)

Solving the system of equations in (48) we get:

Case 1 $\mu = \sqrt{-\frac{K_1}{2\;K_3}}$, $A = \mp\sqrt{\frac{K_1}{K_2}}$ (45)

Case 1 $\mu = \sqrt{-\frac{K_1}{2\ K_3}}$, $A = \mp \sqrt{\frac{3\,\gamma}{K_5}\ \frac{K_1}{K_3}}$ (46)

Case 3 $\mu = \sqrt{-\frac{K_4}{6\,\gamma}}$, $A = \mp \sqrt{\frac{K_4}{K_5}}$ (47)

Case 4 $\mu = \sqrt{-\frac{K_4}{6\,\gamma}}$, $A = \mp \sqrt{\frac{K_3}{K_2}\ \frac{K_4}{3\,\gamma}}$ (48)

Where :

$K_i > 0\ \ for\ i = 1, 2.\ and\ \ K_j < 0\ \ for\ j = 3, 4,5$ (49)

Then :

$p(x,t) = A\ \ tanh(\mu\ \ (x - v\ t + \chi))$ (50)

and

$q(x,t) = e^{i\,\theta(x,t)}\ A\ \ tanh(\mu\ \ (x - v\ t + \chi))$ (51)

Where the phase component of the soliton is given by Eq.(39).

5.3 Singular Soliton

Seeking the solution by sech function method as in (25)

$p(\xi) = \ A\, csch^{\beta}\,(\mu\ \xi)$ (52)

where A represent the amplitudes of the solitons, μ represents the solitons width.

the system of equations in Eqs. (16) and (17) becomes respectively:

$$K_1 csch^{\beta}\,(\mu\ \xi) - K_2\, A^2\ csch^{3\beta}\,(\mu\ \xi) - \ K_3\ \beta\ \mu^2\ \left[(\beta + 1)csch^{\beta+2}\,(\mu\xi) + \beta\ csch^{\beta}\ (\mu\xi)\right]\ = 0 \quad (53)$$

$$K_4\ \ csch^{\beta}\,(\mu\ \xi) - K_5\, A^2\ csch^{3\beta}\,(\mu\ \xi) - 3\ \gamma\ \beta\ \mu^2\left[(\beta + 1)csch^{\beta+2}\ (\mu\xi) + \beta\ csch^{\beta}\ (\mu\xi)\right] = 0 \quad (54)$$

Equating the exponents and the coefficients of each pair of the sech functions we find from Eqsn. (53-54)

$3\beta\ = \beta\ + 2$, then $\beta\ = 1$ (55)

Thus setting coefficients of Equations (53-54) to zero yields set system of equations:

$K_1\ -\ K_3\ \ \mu^2\ \ = 0$

$K_4\ - 3\,\gamma\ \ \mu^2 = 0$

$-2\,K_3\,\mu^2 -\ K_2\,A^2\ \ = 0$

$$- K_5\, A^2 - 6\,\gamma\, \mu^2 = 0 \tag{56}$$

Solving the system of equations in (56) we get:

Case 1 $\mu = \mp \sqrt{\frac{K_1}{K_3}}$, $A = \mp \sqrt{\frac{-2\,K_1}{K_2}}$ (57)

Case 2 $\mu = \mp \sqrt{\frac{K_1}{K_3}}$, $A = \mp \sqrt{\frac{6\,\gamma}{-K_5}\,\frac{K_1}{K_3}}$ (58)

Case 3 $\mu = \mp \sqrt{\frac{K_4}{3\,\gamma}}$, $A = \mp \sqrt{\frac{-2\,K_3}{K_2}\,\frac{K_4}{3\,\gamma}}$ (59)

Case 4 $\mu = \mp \sqrt{\frac{K_4}{3\,\gamma}}$, $A = \mp \sqrt{\frac{2\,K_4}{-K_5}}$ (60)

Where:

$K_i > 0 \;\; for\; i = 1, 3, 4.\; and\;\; K_j < 0 \;\; for\; j = 2, 5$

Then :

$$p(x,t) = A\;\; csch(\mu\;\; (x - v\,t + \chi)) \tag{61}$$

and

$$q(x,t) = e^{i\,\theta(x,t)}\; A\;\; csch(\mu\;\; (x - v\;t + \chi)) \tag{62}$$

Where the phase component of the soliton is given by Eq.(39)

6. CONCLUSION

In this paper the dispersive bright, dark and singular soliton solutions to SHE with Kerr law of nonlinearity was studied. The Sech, Tanh, and Csch function method has been successfully applied to find solitons solutions for the Schrödinger–Hirota equation. Several constraint conditions were assuring the existence of such solitons.

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